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# Active Disk Paxos with infinitely many processes

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Abstract We present an improvement to the Disk Paxos protocol by Gafni and Lamport which utilizes extended functionality and flexibility provided by *Active Disks* and supports unmediated concurrent data access by an unlimited number of processes. The solution facilitates coordination by an infinite number of clients using finite shared memory. It is based on a collection of read-modify-write objects with faults, that emulate a new, reliable shared memory abstraction called a *ranked register*. The required read-modify-write objects are readily available in Active Disks and in Object Storage Device controllers, making our solution suitable for state-of-the-art Storage Area Network (SAN) environments.

**Keywords** Shared memory · Consensus · Paxos · Infinitely many processes · Non-responsive object faults

## **1** Introduction

In this paper we present a solution for universal service replication using a data-centric approach. In this approach, a highly available service is implemented by a replicated set of servers, a threshold of which may be faulty. Each server is responsible solely for implementing certain objects, e.g., a single shared register, that is accessible by any number of clients. Our paradigm provides for coordination and information sharing among transient clients, possibly numerous, through the group of servers. It does not require servers to interact among themselves, and it avoids the complexity of failure monitoring and reconfig-

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uration which is manifested, e.g., in group communication middlewares [12, 46].

The data-centric paradigm also faithfully reflects recent advances in hardware technology that have made possible a new approach for storage sharing, in which clients access disks directly over a storage area network (SAN). In a SAN, disks are directly attached to high speed networks that are accessible to clients. The clients access raw disk data, which is mediated by disk controllers with limited memory and CPU capabilities. Clients run file system services and name servers on top of raw I/O. Since clients (or a group of designated SAN servers) need to coordinate and secure their accesses to disks, they need to implement distributed access control and locking for the disks. However, once a client obtains access to a file, it accesses data directly through the SAN, thus eliminating the slowdown bottleneck at the file system server. IBM's Storage Tank [8] is an example of a commercially available SAN system that solves many of the coordination, sharing and security issues involved with SANs (see Sect. 2 for more examples). In this paper, we tackle the issue of scaling the number of clients that are served by a SAN.

From here on, we refer to shared storage units in our data-centric system simply as *objects*. As in many other distributed settings, a fundamental enabler in this environment for clients to coordinate their actions is an agreement protocol. It is well known that in order to solve agreement in a non-blocking manner three phases are needed [48, 49]. This leads to the usage of the Paxos protocol [18, 34, 35, 37] and its variants, as is done, e.g., in Petal [38] and Frangipani [51]. Briefly, the Paxos protocol is a three-phase commit protocol that uses the 1st phase to determine a proposition value, the 2nd phase to fix a decision value, and the 3rd phase to commit to it. The Paxos protocol was recently adapted for utilization in the shared-memory model in the Disk Paxos protocol [23]. In this work we provide several important contributions that enhance this line of research:

- We provide an adaptation of Paxos that supports infinitely many clients;
- The memory complexity of our solution is constant;

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- Our construction makes use of a modular building block, called a *ranked register*, that promotes understanding and analysis of Paxos and of general coordination in distributed systems;
- Both our agreement protocol and our atomic object emulation are built directly over the ranked register abstraction, providing for each an efficient one-tier implementation. In contrast, most atomic object emulation algorithms found in the literature utilize the Consensus object as a building block.

Of the above contributions, the most tangible one is the extension to support infinitely many clients. Both the original Paxos protocol and its Disk variant are geared toward a fixed and known number of clients. In particular, in Disk Paxos, each client must use a pre-designated memory to write values, and must read the values written by all other potential clients. Consequently, adding new clients to the system is a costly operation that involves real-time locking [23]. Also, the complexity of memory (disk) operations is linear in the number of clients.

In contrast, our solution is ignorant of the number of participating clients and their identities. It builds on a strengthening of the shared memory model. Our use of strong memory objects is justified by the impossibility result of Sect. 7, that shows that even in failure-free runs, finite read/write memory is insufficient for solving agreement among infinitely many processes.<sup>1</sup> Hence, to provide a solution which is realistic in practice, we employ stronger memory objects.

Note that the strengthened memory model is justified in practice. First, servers may support arbitrarily complex object semantics, and as for disks, this approach is motivated by recent development in controller logic that enhances the functionality of disks for SAN and provide for *Active Disks*, capable of supporting stronger semantics objects (see, e.g., [25]). In particular, specialized functions that require specific semantics not normally provided by drives can be provided by remote functions on Active Disks. Examples include a *read-modify-write* operation, or an atomic *create* that both creates a new file object and updates the corresponding directory object. Such advanced operations are already used for optimization of higher-level file systems such as NFS on NASD [26].

The existence of strong shared memory objects does not obviate the need for an agreement protocol. Admittedly, if we had even one reliable read-modify-write object, we could leverage coordination off it to solve agreement, as shown by Herlihy [30]. However, objects stored by servers or disks could become unavailable. Unfortunately, it is impossible to use a collection of fail-prone read-modify-write objects to emulate a reliable one [31]. Hence, our construction is necessarily more involved. It should be noted that using a farm of shared objects also has the benefit beyond high availability. Even in the case that disks are considered reliable, distributing client accesses among multiple objects prevents unnecessary contention. Hence, our solution provides for both high availability, and for load sharing among storage servers.

Our solution first breaks the Paxos protocol using an abstraction of a shared object called a *ranked register*, which is driven by a recent deconstruction of Paxos by Boichat et al. [6]. (We compare our ranked register abstraction with the round-based register of [6] in Sect. 4.) Briefly, a ranked register supports rr-*read* and rr-*write* operations that are both parameterized by a *rank* whose values are taken from a totally ordered set fixed in advance (e.g., the Paxos *ballots* are integers). The main property of this object is that a rr-*read*with rank  $r_1$  is guaranteed to "see" any completed rr-*write* whose rank  $r_2$  satisfies  $r_1 > r_2$ . In order for this property to be satisfied, some lower ranked rr-*write* operations that are invoked after a rr-*read* has returned must *abort*. Armed with this abstract shared object, we show the following two constructions:

- 1. We provide a simple implementation of Paxos-like agreement and universal object emulation using the abstraction of one reliable shared ranked register that supports infinitely many clients. Briefly, in these implementations a participating client chooses a (unique) rank, rr-*read* s the ranked register with it, and then writes the ranked register either with the value it read (if exists) or with its own input. (In case of universal object emulation, the agreement value is the operation prefix, possibly extended by the client's input operation.) If the rr-*write* operation succeeds (i.e., it does not abort), then the process decides on the written value. Else, it retries with a higher rank.
- 2. The reliable shared ranked register abstraction cannot be supported for an unbounded number of clients using only finite read/write memory (proof is provided in Sect. 7). Furthermore, no single fail-prone object may implement it. Therefore, we provide an implementation of a ranked register shared among an unbounded number of clients. The implementation employs a farm of read-modify-write registers, of which a threshold may become non-responsive. The fault tolerant emulation performs each rr-*read* or rr-*write* operation on a majority of the disks, and takes the maximally ranked result as the response from an operation. The number of objects required for the emulation is determined only by the level of desired fault tolerance, regardless of the number of participating clients.

Our approach is readily implementable in a SAN with Active Disks. To this extent, it may serve as an important specification of the kind of functionality that is desired by SAN clients and that disk manufacturers may choose to provide. Additionally, our approach faithfully represents another realistic setting, the classic client-server model, with a potentially very large and dynamic set of clients. This is the setting for which scalable systems like the Fleet object repository [42] were designed. We advocate the datacentric approach in more detail in two recent position papers [14, 41].

<sup>&</sup>lt;sup>1</sup> Section 7 actually provides a stronger result, proving impossibility of constructing a different type of object than a consensus object. By the universality of the consensus object [30], this a fortiori implies impossibility of constructing agreement.

## 2 Related work

Our work deals with solving the Consensus problem [36], one of the most fundamental problems in distributed computing. Consensus is the building block for replication paradigms such as state machine replication [50, 33], group membership (see [12, 46] for survey), virtual synchrony [5], atomic broadcast [11], total ordering of messages [22, 32], etc. Consensus is known to be unsolvable in most realistic models such as asynchronous message passing systems [21] and asynchronous shared memory with read/write registers [19, 30, 40] if even a single process can fail by crashing. While it is usually straightforward to guarantee the consistency of a consensus decision alone (safety), the difficulty is in guaranteeing progress in face of uncertainty regarding process failures. The usual approaches to circumventing Consensus impossibility include strengthening the basic model by assuming different degrees of synchrony (see e.g., [17, 19, 20]), augmenting the system with unreliable failure detectors [11], and employing randomization (see a survey in [16]). Specifically, our solution uses one of the most widely deployed implementations of the state machine replication [33, 50], the Paxos algorithm [18, 34, 35, 37]. At the core of Paxos is a consensus algorithm called Synod. The Synod protocol deals with the Consensus impossibility by guaranteeing progress only when the system is stable so that an accurate leader election is possible. This assumption is equivalent to assuming the  $\Omega$  failure detector of [10] which was shown in [10] to be the weakest failure detector that can be used to solve Consensus.

As shown below in Sect. 7 though, when an infinite number of processes is present, even if they are all nonfaulty, agreement is impossible to achieve using only a finite number of atomic read/write registers. Not surprisingly, the Paxos protocol is in fact designed with built-in knowledge of all of the participants. The focus of our work is on guaranteeing safety of the consensus decision in the presence of an infinite number of processes. Other results in this model and a classification based on levels of simultaneity can be found in [24, 43]. As for liveness, we can use standard approaches as above to circumvent impossibility, and we leave it outside the scope of this work.

Our usage of shared-access SAN disks as shared memory is greatly influenced by the recent Disk Paxos protocol of Gafni and Lamport [23]. In Disk Paxos, the protocol state is replicated at network attached disks some of which can crash or become inaccessible. The participating processes access the state replicas directly over a SAN. Disk Paxos assumes simple commodity disks which support only primitive read and write operations. It supports a bounded and known number of clients, and uses disk memory proportional to their number. In contrast, we stipulate Active Disks that are capable of serving higher semantics objects, which provide us with the strength needed to guarantee safe decisions in face of an unbounded number of clients. The amount of memory we utilize per disk is fixed regardless of the number of participating clients.

The environment model that faithfully reflects our setting is an asynchronous shared memory system where processes interact by means of a finite collection of shared objects some of which can be faulty [1, 31]. Similarly, the Consensus protocol of Disk Paxos, called Disk Synod, is in fact an implementation of Consensus in an asynchronous shared memory system with atomic read/write registers which can incur non-responsive crash failures. It should be noted that in [31], Jayanty et al. prove that it is impossible to implement wait-free Consensus in such an environment if at most one shared object can stop responding forever. This result holds regardless of the number, size and type of the shared objects used by the implementation. Hence, merely by stipulating stronger disks we would still be unable to circumvent the impossibility. Nevertheless, we show that the ranked register is sufficient for implementing non-faulttolerant Consensus with unbounded number of participants. A remarkable feature of the ranked register is that it allows for wait-free implementation in a shared memory system with non-responsive crash faults and therefore, can be used as a building block for implementing fault-tolerant Disk Paxos with unbounded number of processes. As before, the way to guarantee progress despite the impossibility result is by augmenting the system with a leader election primitive which is required to be eventually accurate in order for the protocol to be live.

Our ranked register abstraction was largely inspired by work of Boichat et al. [6] on deconstructing the Paxos protocol. This paper proposes a modular decomposition of Paxos based on a simple shared memory register called roundbased register. Intuitively, both the round-based register and the ranked register encapsulate the notion of Paxos ballots<sup>2</sup> which are used by the protocol to ensure value consistency in presence of concurrent updates. While being in line with the general deconstruction idea of Boichat et al., our ranked register nevertheless provides weaker guarantees and supports a slightly different interface. A detailed comparison is provided in Sect. 4. A different deconstruction of the Paxos protocol is provided in [7]. This deconstruction employs a more abstract shared-object definition, called  $\diamond$  *Register*. The  $\diamond$ Register avoids referring to ranks (or rounds), and thus is a higher level abstraction that does not directly include implementation details. On the other hand, as discussed in [7], the  $\diamond$  *Register* admits inefficient implementations. This is prevented in the specification of our ranked register.

## 2.1 SAN technology

Our work was motivated in part by advances in storage technology and the SAN paradigm. A storage area network enables cost-effective bandwidth scaling by allowing the data to be transferred directly from network attached disks to clients so that the file server bottleneck is eliminated. The Network Attached Secure Disks (NASD) [25] of CMU is

<sup>&</sup>lt;sup>2</sup> Ballots roughly correspond to *rounds* and to *ranks* in the roundbased and the ranked register respectively.

perhaps the most comprehensive joint academy-industry project which laid the technological foundation of network attached storage systems. NASD introduced the notion of an *object storage device (OSD)* which is a network attached disk that exports variable length "objects" instead of fixed size blocks. This move was enabled by recent advances in the Application Specific Integrated Circuit (ASIC) technology that allows for integration sophisticated special-purpose functionality into the disk controllers. The NASD project also addresses other aspects of the network attached disk technology such as file system support [26], security [27] and network protocols [25].

Active Disks [2, 47] is a logical extension of the OSD concept which allows arbitrary application code to be down-loaded and executed on disks. One of the applications of the active disks technology is enhancing disk functionality with specialized methods, such as atomic read-modify-write, that can be used for optimization and concurrency control of higher-level file systems.

Issues concerned with data management in SAN-based file systems, such as synchronization, fault tolerance and security, are investigated in [8] in the context of the IBM Storage Tank project.

Other work which addresses scalability and performance issues of network storage systems (not necessarily concerned with network attached disks) include NSIC's Network-Attached Storage Device project [45], the Netstation project [28] and the Swarm Scalable Storage System [29]. Petal [38] is a project to research highly scalable block-level storage systems. Frangipani [51] is a scalable distributed file system built using Petal.xFS: Serverless Network File Service [4] attempts to provide low latency, high bandwidth access to file system data by distributing the functionality of the server (e.g., cache coherence, locating data, and servicing disk requests) among the clients.

Concurrency control was identified as one of the critical issues in the network attached storage technology because of inherent lack of a central point of coordination [3]. The concurrency control in the Petal [38] virtual disk storage system and the Frangipani [51] file system is achieved using replicated lock servers which utilize Paxos for consistency. Consequently, Disk Paxos is a natural candidate for enabling lock management in network attached storage systems. In this paper we show that by enhancing network attached disk functionality with two simple read-modify-write operations, which are realistic to support with the OSD and Active Disk technologies, it is possible both to adapt Disk Paxos to support an unbounded number of clients and to reduce its communication cost.

#### **3** System model

We consider an asynchronous shared memory system consisting of a countable collection of client processes interacting with each other by means of a finite collection of shared objects. The processes are designated by numbers 1, 2, .... Clients may fail by stopping (crashing). The implementation should be wait-free in the sense that the progress of each non-faulty client should not be prevented by other clients concurrently accessing the memory as well as by failures incurred by other clients. The shared memory objects themselves may be crash faulty. The model above is called in [31] the *non-responsive crash (NR-Crash)* failure model, and we shall use this name here-on.

According to [31], wait-free consensus is impossible in such a setting. This result holds regardless of the number, size and type of the shared objects used by the implementation. Therefore, similar to the Paxos approach, we overcome this impossibility by augmenting the system with a leader oracle. The oracle guarantees the eventual emergence of a unique non-faulty leader, though when this happens is unknown to the clients themselves.

The boolean failure detector oracle we employ, denoted  $\mathcal{L}$ , is as follows. Let  $\mathcal{L}_i$  denote the local instance of  $\mathcal{L}$  at *i*, with a boolean isLeader() operation returning the current value output by  $\mathcal{L}_i$ . Then,  $\mathcal{L}$  is required to satisfy the following property eventually:

Property 1 (Unique Leader) There exists a correct process *i* such that every invocation of  $\mathcal{L}_i$ .isLeader() returns true, and for each process  $j \neq i$ , every invocation of  $\mathcal{L}_j$ .isLeader() returns false.

### 4 The ranked register

Our Consensus and atomic object constructions (see Sect. 5) employ a special type of shared memory register, called a *ranked register*, which for now we assume is failure-free. In Sect. 6, we show how to implement a fault tolerant ranked register.

Intuitively, the ranked register encapsulates the notion of *ballots* which are used by the Paxos protocol to ensure value consistency in presence of concurrent updates. The idea of modeling the Paxos protocol this way is due to [6]. However, while the ranked register interface bears similarities to the *round-based register* of [6], its specification is weaker than that of [6]. We discuss the differences below. The register provides a clean isolation of the essential properties of Paxos into a well-defined building block, thus simplifying reasoning about the protocol behavior.

We now give a formal specification of the ranked register. Let *Ranks* be a totally ordered set of ranks with a distinguished initial rank  $r_0$  such that for each  $r \in Ranks$ ,  $r > r_0$ ; and *Vals* be a set of values with a distinguished initial value  $v_0$ . We also consider the set of pairs denoted *RVals* which is *Ranks* × *Vals* with selectors *rank* and *value*. A ranked register is a multi-reader, multi-writer shared memory register with two operations: rr-*read*(r)<sub>i</sub> by process  $i, r \in Ranks$ , whose corresponding response is  $value(V)_i$ , where  $V \in RVals$ . And rr-*write*(V)<sub>i</sub> by process  $i, V \in RVals$ , whose reply is either *commit*<sub>i</sub> or *abort*<sub>i</sub>. Note that in contrast to a standard read/write register interface, both rr-*read* and rr-*write* operations on a ranked register take a rank as an additional argument; and its **rr**-write operation might abort, whereas the write operation on a standard read/write register always commits (i.e., returns *ack*).

In the following discussion we often say that a rr-read operation R returns a value V meaning that the register responds with value(V) in response to R. We also say that a rr-write operation W commits (aborts) if the register responds with commit (abort) in response to W.

For simplicity, we assume that each run starts with  $W_0 = rr$ -write $(\langle r_0, \bot \rangle)$  which commits. Furthermore, we will restrict our attention to runs in which invocations of rr-write on a ranked register use unique ranks. More formally, we will henceforth assume that all runs satisfy the following:

**Definition 1** We say that a run satisfies rank uniqueness if for every rank  $r \in Ranks$ , it there exists at most one  $v \in Vals$  and one process i such that rr-write $(\langle r, v \rangle)_i$  is invoked in the run.

In practice, rank uniqueness can be easily ensured by choosing ranks based on unique process identifier and a sequence number. The main reason we use this restriction is to simplify establishing the correspondence between the values written with specific ranks and the values returned by the **rr**-read operation.

We now give a formal specification of the ranked register. We start by introducing the following definition:

**Definition 2** We say that a rr-read operation R = rr-read $(r_2)_i$  sees a rr-write operation W = rr-write $(\langle r_1, v \rangle)_j$  if R returns  $\langle r', v' \rangle$  where  $r' \ge r_1$ .

The ranked register is required to satisfy the following three properties:

Property 2 (Safety) Every rr-read operation returns a value and rank that was written in some rr-write invocation. Additionally, let W = rr-write( $\langle r_1, v \rangle$ )<sub>i</sub> be a rr-write operation that commits, and let R = rr-read( $r_2$ )<sub>j</sub>, such that  $r_2 > r_1$ . Then R sees W.

Property 3 (Non-Triviality) If a rr-write operation W invoked with the rank  $r_1$  aborts, then there exists a rr-read (rr-write) operation with rank  $r_2 > r_1$  which is invoked before W returns.

*Property 4 (Liveness)* If an operation (**rr***-read* or **rr***-write*) is invoked by a non-faulty process, then it eventually returns.

Allowing **rr**-write to abort sometimes is crucial for its implementability. Suppose that a **rr**-read operation with rank r returns a value written by a **rr**-write operation with a rank r' < r. Later, when a subsequent **rr**-write with a rank r' < r'' < r is invoked, it must abort due to this **rr**-read.

Also note that our ranked register specification is very weak: In particular, it allows in some situations for rr-*write* operation to commit even though there exists another previously committed rr-*write* with a higher rank. The reason for that not being a problem stems from the way the ranked register is used by the Consensus implementation in Sect. 5.1.

In particular, each process in our Consensus implementation invokes **rr**-*write* only after it invokes **rr**-*read* with the same rank and this **rr**-*read* returns. Thus, the ranked register Safety property ensures that in every finite execution prefix, each value written by a committed **rr**-*write* must be returned by one of the **rr**-*read* operations with a higher rank if such exist. Consequently, in each run of the Consensus implementation, any **rr**-*write* operation, which is invoked after **rr**-*read* with a higher rank has returned, would necessarily abort.

Our specification of ranked register is weaker compared with the round-based register of [6]. The round-based register uses notions of partial operation ordering in the definition of the *write-commit* property ("if write (k, v) commits, and no subsequent write(k', v') with  $k' \geq k$  and  $v' \neq v$ is invoked, then any read(k'') that commits, commits with v if k'' > k'', stressed text added here for clarity). To see that this definition is too strong, consider the following scenario. Suppose that a write  $w_1 = write(k_1, v_1)$  is invoked and is still in progress when another write is invoked,  $w_2 = write(k_2, v_2)$ , with  $k_1 > k_2, v_1 \neq v_2$ . In this case,  $w_2$  may commit. However, a subsequent read may "see"  $w_1$ , and the value of  $w_1$  may be returned, contradictory to the requirement. In fact, the distributed implementation of round-based register in [6] does not prevent this. Moreover, it does not seem possible to prevent this in our setting. Finally, we should note that just by dropping 'subsequent' from the specification results in different problems. It is our view that there is no easy way to form the rankedregister specification using operation ordering, and hence, the specification above is qualitatively different from that of the round-based register.

## 5 Consensus and atomic object emulation

In this section we present the implementations of Consensus and of an arbitrary typed atomic object (a universal construction) based on the ranked register abstraction defined in the previous section. The algorithms in this section use the ranked register as a black box.

In addition to a shared ranked register, our algorithms also employ atomic shared registers. It should be noted that these objects can be implemented in our models in a similar manner to the ranked-register implementation, and hence we omit their explicit constructions.

#### 5.1 Consensus using a ranked register

We now outline an agreement protocol which employs a shared ranked register. The pseudocode of the Consensus implementation is depicted in Fig. 1. Each process *i* iterates through the following steps until the decision is reached: First, *i* checks whether some process has decided and written the agreement value into the *decision* register. If yes, this value is returned. Otherwise, *i* calls a local procedure, chooseRank which is assumed to output monotonically increasing values  $r \in Ranks$ , and then waits until the output

Shared: Ranked registers rr initialized by rr-write $(\langle r_0, \perp \rangle)$  which commits; Regular register *decision*, with values in *RVals*, initialized by  $write(\langle r_0, \perp \rangle)$ Local:  $V \in RVals \cup \{abort\}, r \in Ranks;$ 

```
Process i:
propose(v), Vals \rightarrow Vals
      r \leftarrow r_0;
      while (true) do
            V \leftarrow decision.read();
            if (V.value \neq \bot)
                   return V.value;
            if (\mathcal{L}_i.isLeader()) then
                   r \leftarrow chooseRank(r):
                   V \leftarrow \text{DECIDE}(\langle r, v \rangle);
                  if (V \neq abort)
                         return V.value;
            fi
      od
Function \text{DECIDE}(\langle r, v \rangle), RVals \rightarrow RVals \cup \{abort\}:
      V \leftarrow rr.rr-read(r)_i;
      if (V.value = \bot) then
            V.value \leftarrow v;
      V.rank \leftarrow r;
      if (rr.rr-write(V)_i = commit) then
            decision.write(V);
            return V;
      fi
      return abort;
```

Fig. 1 Consensus using a ranked register

of  $\mathcal{L}_i$  becomes *true*. Once this happens, the local DECIDE routine is invoked. It takes as arguments *i*'s initial value and the chosen rank. It returns the agreement value or aborts. The DECIDE routine is guaranteed to return an agreement value at the latest when a non-faulty leader has been elected and allowed to force a decision (i.e., Property 1 holds). We now outline the correctness argument of the agreement algorithm. Recall that  $W_0$  is an initialization rr-write operation, assumed to commit at the start of any execution. Ignoring this initialization, the next lemma shows that once a consensus value commits, it remains fixed as the decision value throughout the execution.

**Lemma 1** For any finite execution  $\alpha$ , let  $W_1 = rr.\mathbf{rr}$ -write $(\langle r_1, v_1 \rangle), W_1 \neq W_0$  be the lowest ranked  $\mathbf{rr}$ -write invocation which commits in  $\alpha$ . Then, in any extension of  $\alpha$  in which  $W = rr.\mathbf{rr}$ -write $(\langle r, v \rangle), r > r_1$ , is invoked,  $v = v_1$ .

*Proof* Our proof strategy is to build a chain of rr-write's from  $W_1$  to W, such that each W writes the value that it reads from the preceding rr-write in the chain. We then show that the same value is written in all of these rr-write's by induction on the length of such chains.

Indeed, let R = rr.rr-read(r) be the rr-read corresponding to W that is executed before W is invoked. By safety, R returns the pair  $\langle r_1, w_1 \rangle$  or a higher ranking pair  $\langle r_k, w_k \rangle$  that was written in some  $W_k = rr-write(r_k, w_k)$ . Since  $r_k > r_1$ , again the corresponding  $rr.rr-read(r_k)$  returns  $\langle r_1, w_1 \rangle$  or a higher ranked written value. And so on. Eventually, we obtain a unique chain  $W_1, W_2, \ldots, W_k, W$ , such that for each of  $W_2, \ldots, W_k, W$ , the corresponding **rr***-read* returns the value/rank pair written by the preceding **rr***-write* in the chain.

We now show by induction on the length k of the chain that W writes  $v_1$ . If k = 1, then R returns  $v_1$  and by the agreement protocol W writes  $v_1$ .

Otherwise, suppose for all chains of length  $\langle k$  it holds that the last rr-*write* writes  $v_1$ , and consider the chain above of length k. For  $W_k$ , the (unique) chain from  $W_1$  is  $W_1, W_2, \ldots, W_k$ . By the induction hypothesis,  $W_k$  writes  $v_1$ . Hence, again R reads  $v_1$  and according to the protocol, W writes  $v_1$ .

The following theorem immediately follows from Lemma 1 (and the protocol):

**Theorem 1** The algorithm in Fig. 1 guarantees that for any two processes i and j such that  $propose(v)_i$  returns V and  $propose(v')_j$  returns V', V = V'; and V is the argument of some propose operation which was invoked in the run.

Next, we show liveness.

**Theorem 2** If some correct process invokes propose, then eventually all correct processes decide.

*Proof* First note that the regular register semantics imply that once some process decides and completes its write operation to the *decision* register, all other process will eventually read this value and decide.

Otherwise, by definition of  $\mathcal{L}$ , there exists time T such that Property 1 holds at all times t > T. Assume that *decision* is not written before T. Since by the theorem precondition, at least one correct process is taking steps after T, Property 1 implies that there exists a correct process isuch that at all times t > T,  $\mathcal{L}_i$ .isLeader() returns true, and for all  $j \neq i$ ,  $\mathcal{L}_j$ .isLeader() returns false. By nontriviality of the ranked register, rr-write is guaranteed to commit once it is called with a rank which is the highest among all the ranks ever chosen by any process in the system. Since chooseRank returns monotonically increasing ranks, such a rank is eventually returned by chooseRank at *i*. Once, **rr**-write commits, *i* writes the committed value to the *decision* register. Once this happens, all the correct processes eventually decide. 

#### 5.2 Atomic object emulation using a ranked register

Ultimately, the purpose of forming coordination is to support data sharing among clients consistently. Many protocols leverage atomic data emulation off of the consensus building block we already have. In this section we show how a ranked register can be used *directly* to construct an atomic object of an arbitrary type T. This yields a one-tier, practical construction.

An object type  $\mathcal{T}$  is a tuple  $\langle \sum, OP, Res, G \rangle$ , where  $\sum$  is a set of the type states, OP is a set of operations, Res is a set of responses, and G is a sequential specification of  $\mathcal{T}$ 

Types: OpSeq is a sequence over  $OP \times Res$ ;  $RVals = Ranks \times OpSeq$  with selectors rank and prefix; Shared: A ranked register rr with values in *RVals* initialized by  $rr.rr-write(\langle r_0, \langle \rangle))$  which commits. Local:  $r \in Ranks$ , initially  $r = r_0$ ;  $V \in RVals$ ; Process i:  $submit(op)_i: OP \rightarrow Res:$  $r \leftarrow r_0;$ while(true) do if  $(\mathcal{L}_i.isLeader())$  then  $r \leftarrow chooseRank(r);$  $V \leftarrow \text{ORDER}(r, op);$ if  $(V \neq abort)$ return res :  $\exists i \langle op, res \rangle = V.prefix[i];$ fi od Function ORDER(r, op), Ranks  $\times OP \rightarrow RVals \cup \{abort\}$  $V \leftarrow rr.rr-read(r)$ : if  $(\forall i \forall res : V.prefix[i] \neq \langle op, res \rangle)$  $V.prefix \leftarrow APPLY(V.prefix, op);$  $V.rank \leftarrow r;$ if (rr.rr-write(V) = commit) then return V; else return *abort*; fi

Fig. 2 Emulating an arbitrary atomic object using a ranked register

which maps the pairs in  $OP \times \sum$  to the pairs in  $Res \times \sum$ . Let  $\sigma, \sigma' \in \sum, op \in OP$  and  $res \in Res$ . We say that  $\langle \sigma', res \rangle$  is the result of *applying op* to  $\sigma$  iff  $\langle \sigma', res \rangle = G(\sigma, op)$ .

The atomic object emulation pseudocode appears in Fig. 2. The operation submit takes as a parameter an operation to execute, and returns the operation response. For simplicity, we assume that each  $op \in OP$  can be submitted at most once throughout the run. In practice, this requirement can be easily enforced by assigning each newly submitted operation a unique id which, e.g., can be the pair consisting of the process id and a counter. We assume that the sequential specification of  $\mathcal{T}$ , G as well as the initial state  $\sigma_0$  of the type  $\mathcal{T}$  instance being emulated is known to all the participating processes. As before, we assume that the chooseRank routine returns unique and monotonically increasing ranks. The ranked register is used to build a common invocation sequence which, when applied to the object states starting from  $\sigma_0$ , ensures that the returned responses are consistent with G. The set of values which are written to and read from the ranked register are taken from the set of all sequences over the set  $OP \times Res$ . We denote by APPLY(seq, op), where seq is a sequence over  $OP \times Res$  and  $op \in OP$ , the sequence obtained by appending  $\langle op, res \rangle$  to seq, where res is the result of applying op to the final state reached when applying the operations in *seq* from the initial state  $\sigma_0$ .

We now set off to show the emulation algorithm correct. The next lemma is similar to Lemma 1 of the Consensus correctness argument: **Lemma 2** For any execution  $\alpha$ , let  $W_0 = \operatorname{rr-write}(\langle \pi_0, r_0 \rangle)$ be a rr-write operation which commits. Then, in any extension of  $\alpha$  in which  $W = \operatorname{rr-write}(\langle \pi, r \rangle)$ ,  $r > r_0$  is invoked,  $\pi_0$  is a prefix of  $\pi$ .

*Proof* By the same argument as in the proof of Lemma 1, there exists a unique chain  $W_0, W_1, \ldots, W_k, W$ , such that for each  $W_1, \ldots, W_k, W$ , the corresponding rr-*read* returns the prefix/rank pair written by the preceding rr-*write* in the chain.

We now show by induction on the length k of the chain that  $\pi$  is an extension of  $\pi_0$ . If k = 0, then the rr-*read* preceding W reads  $\pi_0$ , and by the ORDER routine, W either writes  $\pi_0$ , or appends an operation/response pair to  $\pi_0$ , and then writes the resulting sequence.

Otherwise, suppose for all chains of length l < k, it holds the last rr-write writes a sequence  $\pi_l$  which extends  $\pi_0$ , and consider the chain above the length k. For  $W_k$ , the (unique) chain from  $W_0$  is  $W_0, W_1, \ldots, W_k$ . By the induction hypothesis,  $W_k$  writes a sequence  $\pi_k$  which extends  $\pi_0$ . Hence, again the rr-read operation preceding W reads  $\pi_k$ , and according to the ORDER code, W writes a sequence  $\pi$ which is either equal or extends the sequence  $\pi_k$ .

**Theorem 3 (Atomicity)** *The algorithm in Fig. 2 emulates an atomic object of type* T.

*Proof* By Lemma 2, all the operation prefixes written by the committed **rr**-write invocations form a sequence  $\bar{\pi} = \pi_0, \pi_1, \ldots, \pi_k$  such that (1)  $\pi_0 = \langle \rangle$ ; and for each i > 0: (a)  $\pi_i$  either equal to or extends  $\pi_{i-1}$ ; and (b) the rank of

rr-write which wrote  $\pi_i$  is higher than the rank of rr-write which wrote  $\pi_{i-1}$ . Since according to the algorithm, the operation result is obtained by applying all the operations in a committed prefix to the initial object state, the returned results are consistent with  $\bar{\pi}$ . Moreover, since no lower ranked rr-write can commit if it is invoked after a higher ranked rr-write has committed,  $\bar{\pi}$  preserves the temporal order of non-concurrent rr-write operations.

Finally, the next theorem asserts the liveness:

**Theorem 4 (Liveness)** If a correct process i invokes  $submit(op)_i$ , then there exists a process j such that  $submit(op)_j$  eventually returns.

*Proof* The proof is based on the same argument as the proof of Theorem 2.  $\Box$ 

The liveness property above is notably weak. In particular, it does not guarantee that every **submit** operation terminates, but rather, provides a global guarantee of progress; singular operations could in principle be starved. The usual approach to transform this guarantee into a proper liveness provision is to establish a set-object into which pending operations are thrown. Each leader should then help set an order on any operation in the set, and thus, eventually all operations are ordered. We omit the details of this mechanism for brevity.

## 6 Implementing a ranked register

In this section, we deal with the problem of implementing a wait-free shared ranked register. First, in Sect. 6.1, we specify how a single ranked register is implemented from a read-modify-write object. Second, in Sect. 6.2, we present a wait-free self-construction of the ranked register for the NR-Crash failure model.

### 6.1 A single ranked register

Our shared memory model assumes the existence of atomic shared objects such as read-modify-write registers. By this, we capture the assumption that each "disk" is capable of accepting from clients subroutines with I/O operations for execution, and indivisibly performing them. The disk itself may become unavailable, and hence, the shared memory objects it provides may suffer non-responsive crash faults. For this reason, no single read-modify-write object suffices for solving agreement on its own (as in Herlihy's consensus hierarchy, see [30]). Rather, we first use each read-modify-write object to construct a ranked-register (which may also incur a non-responsive crash fault), and then, use a collection of ranked registers to construct a non-faulty ranked-register, from which agreement is built.

Let  $X = (Ranks \times Ranks \times Vals) \cup \{\langle r_0, r_0, \bot \rangle\}$  with selectors rR, wR and val. The implementation of a ranked register uses a single read-modify-write shared object  $x \in X$ of unbounded size whose field x.rR holds the maximum rank with which a rr-*read* operation has been invoked; x.wRholds the maximum rank with which a rr-*write* operation has been invoked; and x.val holds the current register value. The implementation pseudocode is depicted in Fig. 3. It is quite straight-forward: read returns the current value of the register, and records its own rank. Write checks wether a higher ranking read was invoked, aborts if yes, and if not, modifies the value of the register and records its own rank. For clarity, invocations of read-modify-write operations rmw-*read* and

```
Types: X = (Ranks \times Ranks \times Vals) \cup \{\langle r_0, r_0, \bot \rangle\} with selectors rR, wR and val
Shared: x \in X.
Initially x = \langle r_0, r_0, \bot \rangle
Local: V \in RVals, status \in \{ack, nack\}.
Process i:
rr-read(r)_i:
                                                                          Read-modify-write procedures:
        lock x:
                                                                          rmw-read(r):
              V \leftarrow \mathsf{rmw}\text{-}read(r)
                                                                                if (x \cdot rR < r)
        unlock x
                                                                                      x.rR \leftarrow r
        return V
                                                                                return \langle x.wR, x.val \rangle
\operatorname{rr-write}(\langle r, v \rangle)_i:
                                                                          \mathsf{rmw}-write(r, v):
                                                                                if (x.rR \le r \land x.wR < r)
        lock x:
                                                                                      x.wR \leftarrow r
              status \leftarrow \mathsf{rmw}\text{-}write(r, v)
        unlock x
                                                                                      x.val \leftarrow v
        if (status = ack)
                                                                                      return ack
              return commit
                                                                                return nack
        return abort
```

rmw-*write* are enclosed within "lock" and "unlock" statements, to indicate that they execute indivisibly.

**Lemma 3** The pseudocode in Fig. 3 satisfies Safety.

*Proof* That a rr-*read* operation can only return a valid value that was actually used in a rr-write operation or  $(r_0, \perp)$ is obvious from the code. Now consider a rr-write operation  $W_1 = \operatorname{rr-write}(\langle r_1, v_1 \rangle)_i$  that commits and let  $R_2 =$ rr-read $(r_2)_i, r_2 > r_1$  be a rr-read operation which returns  $\langle r, v \rangle$ . Let  $mw_1$  denote the rmw-write() procedure called from within  $W_1$  and  $mr_2$  the rmw-read() procedure invoked within  $R_2$ . Since the read-modify-write semantics of x ensures sequential access,  $mr_2$  must be sequenced after  $mw_1$ . For otherwise,  $x \cdot rR \ge r_2 > r_1$  so that  $mw_1$  returns *nack* and  $W_1$  aborts. Thus,  $R_2$  returns the tuple written by a rmw-write procedure mw' which is either  $mw_1$  or some rmw-write procedure sequenced after  $mw_1$ . Let r', v' be the arguments passed to mw'. Then,  $r' \ge r_1$ , since otherwise,  $x.wR \ge r_1 > r'$  so that the value of x remains unchanged. Moreover, by the rank-uniqueness assumption,  $r' = r_1$  implies that  $mw' = mw_1$ . Therefore,  $\langle r, v \rangle = \langle r', v' \rangle$  and either  $\langle r', v' \rangle = \langle r_1, v_1 \rangle$ , or  $r' > r_1$  as needed. 

Lemma 4 The pseudocode in Fig. 3 satisfies Non-Triviality.

*Proof* According to the pseudocode, a rr-write operation W with rank r aborts if the rmw-write() procedure w called within W returns nack. This happens if w sees x.rR > r or  $x.wR \ge r$ . This is only possible if some rmw-write() procedure with rank  $r' \ge r$ , or a rmw-read() procedure with rank r' > r is sequenced before w. This could happen only as a result of some previously returned or concurrent rr-read

Shared: Ranked registers  $rr_j$ ,  $1 \le j \le n$ 

(rr-write) with rank r' > r ( $r' \ge r$ ). By the rank-uniqueness assumption, no two rr-write operations are ever invoked with the same rank. Therefore, W can abort only due to some previously returned or concurrent rr-read or rr-write with rank r' > r as needed.

**Lemma 5** The pseudocode in Fig. 3 satisfies Liveness.

*Proof* Liveness trivially holds since both rr-*read* and rr-*write* always return something (i.e., the implementation is *wait-free*).

We have proven the following theorem:

**Theorem 5** *The pseudocode in Fig. 3 is an implementation of a ranked register.* 

6.2 A fault-tolerant construction of a ranked register for NR-Crash

In this section we present a wait-free implementation of a ranked register from ranked registers that may experience non-responsive crash faults. The register supports an unbounded number of clients. Our construction utilizes *n* shared ranked registers up to  $\lfloor (n-1)/2 \rfloor$  of which can incur non-responsive crash. The pseudocode appears in Fig. 4. This construction is also straight-forward: Reading and writing are both done at a majority of the ranked registers. As for rr-write, if any of the ranked registers which are accessed returns *abort*, the operation aborts.

## Lemma 6 The pseudocode in Fig. 4 satisfies Safety.

*Proof* That a **rr***-read* operation can only return a valid value that was actually used in a **rr***-write* operation or  $\langle r_0, \bot \rangle$ 

Local: Multisets  $S_1 \subseteq RVals$ ,  $S_2 \subseteq \{commit, abort\}$ . Process i:  $rr-read(r)_i$ :  $S_1 \leftarrow \emptyset$ Invoke  $rr-read(r)_i$  in parallel on the ranked registers in  $\{rr_j\}_{1...n}$ ; Accumulate responses in  $S_1$ ; wait until  $|S_1| \ge \lceil (n+1)/2 \rceil$ ;  $\langle r, v \rangle \leftarrow \langle r', v' \rangle : \langle r', v' \rangle \in S_1 \land r' = max_{\langle r'', v'' \rangle \in S_1} r''$ return  $\langle r, v \rangle$   $rr-write(\langle r, v \rangle)_i$ :  $S_2 \leftarrow \emptyset$ Invoke  $rr-write(\langle r, v \rangle)_i$  in parallel on the ranked registers in  $\{rr_j\}_{1...n}$ ; Accumulate responses in  $S_2$ ; wait until  $|S_2| \ge \lceil (n+1)/2 \rceil$ ; if  $(abort \in S_2)$ return abortreturn commit

Fig. 4 A fault-tolerant ranked register construction for NR-Crash

is obvious from the code. Now consider a rr-write operation  $W_1 = rr$ -write $(\langle r_1, v_1 \rangle)_i$  that commits and let  $R_2 =$ rr-read $(r_2)_j, r_2 > r_1$  be a rr-read operation which returns  $\langle r, v \rangle$ . Since both  $W_1$  and  $R_2$  access at least  $\lceil (n + 1)/2 \rceil$ ranked registers, there exists a single register  $rr_k$  accessed by both  $W_1$  and  $R_2$ . Moreover, the Safety of  $rr_k$  ensures that the tuple  $\langle r', v' \rangle$  returned by  $rr_k$ .rr-read $(r_2)_i$  must satisfy  $r' \ge r_1$ . Since  $R_2$  returns the tuple with maximum rank,  $r \ge r' \ge r_1$  as needed.

# Lemma 7 The pseudocode in Fig. 4 satisfies Non-Triviality.

*Proof* According to the protocol, a **rr**-write operation  $W = \mathbf{rr}$ -write $(\langle r, v \rangle)_i$  aborts if there exists k such that  $rr_k.\mathbf{rr}$ -write $(\langle r, v \rangle)_i$  aborts. By the Non-Triviality of  $rr_k$ , this can happen only if some invocation  $rr_k.\mathbf{rr}$ -write $(\langle r', v' \rangle)_j$   $(rr_k.\mathbf{rr}$ -read $(r')_j)$  with r' > r occur before or concurrently to  $rr_k.\mathbf{rr}$ -write $(\langle r, v \rangle)_i$ . This can only be the case if some **rr**-write or **rr**-read operation with rank r' has been completed before or is concurrent to W.

# Lemma 8 The pseudocode in Fig. 4 satisfies Liveness.

**Proof** Each **rr**-write or **rr**-read operation is guaranteed to terminate since at most  $\lceil (n + 1)/2 \rceil$  ranked registers are required to respond, no more than  $\lfloor (n - 1)/2 \rfloor$  ranked registers can incur non-responsive crash, and each individual non-faulty ranked register is wait-free.

## We have proven the following theorem:

**Theorem 6** The pseudocode in Fig. 4 is a wait-free construction of a ranked register out of n ranked registers such that at most  $\lfloor (n - 1)/2 \rfloor$  can incur non-responsive crash faults.

## 7 Impossibility of constructing ranked-register from read/write registers

Our construction of a fault tolerant ranked-register requires strong (read-modify-write) base objects. In this section we briefly address the natural question of whether this strong memory model is necessary. We prove that a ranked register cannot be implemented using a bounded number of atomic read/write registers (of unbounded size) in the presence of unbounded number of clients, even if clients are failure-free. The main result of this section is expressed in Theorem 7 below. It shows that any algorithm that implements the ranked register specification in a shared memory system with n processes must use at least n atomic read/write registers. It then follows that if the number of processes is not bounded, the number of shared read/write registers needed to implement the ranked register is also unbounded.

In order to prove this result, we utilize the technique of [9] to prove lower bounds on the number of atomic registers needed to solve mutual exclusion. Though the proof technique below is standard, it should be noted that there is no known direct reduction from mutual exclusion to a ranked register, and hence, the results of [9] do not apply directly to the impossibility of constructing a ranked register. In fact, we conjecture that the ranked register is strictly weaker than the mutual exclusion problem, and hence, that no such reduction is possible.

We start with some definitions. We say that two system states s and s' are indistinguishable to process i, denoted  $s \sim s'$ , if the state of process i and the values of all shared variables are the same in s and s'. We say that process i covers shared variable x in system state s if i is about to write on x in s.

**Lemma 9** Suppose that there exists an algorithm that implements a ranked register using only shared atomic read/write registers. Let s be a reachable system state in which r is the highest rank that appears in any operation. Then a **rr**-write operation  $W = \mathbf{rr}$ -write $(\langle r', v' \rangle)_i$  by process i with r' > r must write some shared variable which is not covered in s.

*Proof* Assume in contradiction that no non-covered shared variable is written by i in the course of W. We construct a system execution which violates the Safety property of the ranked register as follows:

We first run from *s* each process which covers some shared variable exactly one step so that they write the shared variables they cover. Let s' be the resulting system state.

Next, we construct an execution fragment  $\alpha_1$  starting in s' and not involving i by invoking a rr-read(r'') operation R at some process  $j \neq i$  whose rank r'' satisfies r'' > r'. By the Liveness and the Safety properties of the ranked register, R must return a value written by some rr-write operation with rank at most r.

We now construct another execution fragment  $\alpha_2$  which starts from *s* as follows: We run *i* solo until *W* commits; since no higher rank appears in *s*, by the Non-Triviality property *W* must indeed commit. By assumption, it writes only shared variables that are covered in *s*. From the resulting state, we run each process which covers some shared variable exactly one step so they overwrite everything written by *i* in its solo run. Let *s''* be the resulting state. Since  $s'' \stackrel{>}{\sim} s'$  for all  $j \neq i$ , we can extend  $\alpha_2$  by running  $\alpha_1$  from *s''*.

By the Safety property of the ranked register, the rr-read operation R must return the value written by W in this execution. However, it returns a value written by a rr-write operation with rank at most r thus violating safety. A contradiction.

We now set off to prove the lower bound. We use the following strategy: We first prove using Lemma 10 that with any algorithm implementing the ranked register for  $n \ge 1$  processes, it is possible to bring the system to a state where at least n - 1 shared variables are covered while running only n - 1 processes. In this state we invoke a rr-write operation whose rank is higher than the the rank of every operation invoked so far. Since this rr-write operation must commit (Non-Triviality), by Lemma 9, it must write to some shared variable which has not been covered yet. This implies that another shared variable is needed in addition to the n - 1 covered ones.

**Lemma 10** Suppose that there exists an algorithm that implements a ranked register for  $n \ge 1$  processes using only shared atomic read/write registers. Let *s* be any reachable system state. Then for any  $k, 1 \le k \le n - 1$ , there exists a state  $s_k$  which is reachable from *s* using steps of processes  $1 \dots k$  only, such that at least *k* distinct variables are covered in  $s_k$ .

## *Proof* The proof is by induction on *k*.

*Basis:* k = 1. Let *s* be any system state. We first run process 1 until it returns from the last operation invoked on 1, if any. This must happen due to the Liveness property of the ranked register. Let *t* be the resulting system state.

In t, we let process 1 invoke a rr-write operation W whose rank is higher than the ranks of all operations invoked so far. By Non-Triviality, W must commit. By Lemma 9, W must write some shared variable which is not covered in state s. We then run 1 until it covers this variable. The resulting state  $s_1$  satisfies the lemma requirements.

Inductive step: Suppose the lemma holds for k, where  $1 \le k \le n - 2$ . Let us prove it for k + 1. Using the induction hypothesis, we run k processes from s until the state  $s_k$  is reached where at least k distinct shared variables are covered. Starting in  $s_k$ , Starting in t, we run process k + 1 until the last operation invoked on k + 1 returns. This must happen due to Liveness. Let t be the resulting state.

In *t* we let process k + 1 invoke a **rr**-write operation *W* whose rank is higher than the ranks of all operations invoked so far. By Non-Triviality, *W* must commit. Moreover, by Lemma 9, *W* must write some shared variable which is not covered in  $s_k$ . So we run k + 1 until it covers this shared variable. The resulting state  $s_{k+1}$  satisfies the lemma requirements.

We are now ready to prove the main theorem:

**Theorem 7** If there exists an algorithm that implements a ranked register for  $n \ge 1$  processes, then it must use at least *n* shared atomic read/write registers.

*Proof* Assume in contradiction that there exists an algorithm which implements a ranked register for  $n \ge 1$  processes using n - 1 shared read/write registers.

Let *s* be the initial system state. Note that there are no covered variables in *s*. We use the result of Lemma 10 and run n - 1 processes from *s* until the state  $s_{n-1}$  is reached where the processes cover n - 1 distinct shared variables. We then invoke a rr-write operation *W* on process *n* whose rank is higher than the ranks of all operations invoked so far. By Non-Triviality, *W* must commit. By Lemma 9, *W* must write some shared variable which is not covered in  $s_{n-1}$ . However, all n-1 shared variables are covered in  $s_{n-1}$ . A contradiction.

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